How Georgi-Jarlskog and SUSY-SO(10) imply a measurable rate for $\mu \to e\gamma$

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Slepton mass matrices have been analyzed in an SO(10) SUSY-GUT, with soft-breaking terms generated at Planck scale. Higher dimensional operators consistent with 4-d string constructions are used in order to generate a Georgi-Jarlskog (G-J) Yukawa texture at $M_{\rm GUT}$. Radiative corrections between $M_{\rm Pl}$ and $M_{\rm GUT}$ generate a substantial non-universality, in the $\tilde{\mu} - \tilde{e}$ sector. This non-universality originates in the flavor dependence of the Higgs assignments required for the G-J texture, and is *unrelated to the large top Yukawa*. The resulting branching ratio for $\mu \to e\gamma$ could make this process observable for large sectors of the MSSM parameter space, with a factor of 10 improvement in statistics.

1. INTRODUCTION

Supersymmetric Grand Unified Theories (SUSY-GUTs), in combination with phenomenologically acceptable *ansatze* about Yukawa structures at GUT, can provide some interesting predictions for low energy CKM parameters. A second (logically disconnected) feature of SUSY-GUTs emerges when soft terms are generated at the Planck scale through the coupling to a hidden sector via supergravity: even if these soft-breaking terms are universal at $M_{\rm Pl}$, radiative corrections between $M_{\rm Pl}$ and $M_{\rm GUT}$ may induce enough flavor dependence in the soft-breaking structure at GUT to be reflected in important low-energy flavor violation [1,2].

Heretofore, these corrections have been almost exclusively reflective of the large top quark coupling – this has differentiated the generations. This talk will contain a synopsis of some recent work [3] which couples the two features in the last paragraph in a fashion leading to a new source flavor violation in the lepton sector. The argument, briefly, is that phenomenologically acceptable fermion textures at GUT (such as G-J) are compatible with specific flavor dependence in the Higgs assignments above $M_{\rm GUT}$; these in turn generate non-universality in the soft structure at GUT, which has phenomenologically significant consequences for the process $\mu \to e\gamma$. These consequences are entirely disconnected from 3rd generation dynamics, and are the core subject of this talk. Full details are given in the published work [3].

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2. THE MODEL

The fermion mass matrices at GUT which incorporate the G-J texture are

$$h^u = \begin{pmatrix} 0 & C & 0 \\ C & 0 & B \\ 0 & B & A \end{pmatrix}, \quad h^d = \begin{pmatrix} 0 & F & 0 \\ F & E & 0 \\ 0 & 0 & D \end{pmatrix},$$
$$h^e = \begin{pmatrix} 0 & F & 0 \\ F & -3E & 0 \\ 0 & 0 & D \end{pmatrix} . \tag{1}$$

The 2-2 entries in h^d, h^e allow for a reasonable low energy value of m_s/m_μ. In SO(10) SUSY-GUTs, a $\mathbf{16}_2\mathbf{16}_2\overline{\mathbf{126}}$ term in the superpotential will provide for the G-J texture. However, a $\overline{\mathbf{126}}$ is disallowed in 4-d string constructions[4], and leads to a rapid breakdown of perturbation theory above GUT. Hence, we are led to use a Yukawa structure containing composite operators which will substitute for the $\overline{\mathbf{126}}$. These operators can be pictured as the result of integrating out some set of heavy superfields at a scale $M \lesssim M_{\rm Pl}$. For

definiteness, the model used will be the one proposed by Babu and Mohapatra[5]with a Yukawa superpotential

$$\begin{aligned} W_{yuk} &= M^{-1}\mathbf{16}_3\mathbf{16}_3(Y_{33}\mathbf{10}_d S_1 + h_{33}\mathbf{10}_u S_2) \\ &+ M^{-1}\mathbf{16}_2\mathbf{16}_3\ (h_{23}\mathbf{10}_u S_3) \\ &+ M^{-2}\mathbf{16}_2\mathbf{16}_2\ (Y_{22}\mathbf{45}_1\mathbf{45}_2\mathbf{10}_d) \\ &+ M^{-3}\mathbf{16}_1\mathbf{16}_2(Y_{12}\mathbf{10}_d S_2^3 + h_{12}\mathbf{10}_u S_1^3) \end{aligned} \tag{2}$$

There are discrete symmetries which prescribe the couplings shown (including those of the gauge singlets $S_{1,2,3}$), and both these and the SO(10) gauge symmetry are broken at $M_{\rm GUT}$. All vevs (except for $\langle S_2\rangle \sim M$) are of order $M_{\rm GUT}$, which sets up the proper Yukawa hierarchy. (All the coupling constants are of $O(1)$.) The vev structure of the $\mathbf{45}$'s ($\langle \mathbf{45}_1\rangle\,||(B-L)$, $\langle \mathbf{45}_2\rangle\,||\ T_{3R}$), leads to an effective $\overline{\mathbf{126}}$ in the product $\langle \mathbf{45}_1\mathbf{45}_2\mathbf{10}_d\rangle$. As an example, the Yukawa coupling B in Eq. (1) is given by $h_{23}\langle S_3\rangle/M$.

3. SOFT-BREAKING TERMS

In this work we assume universal soft scalar masses m_0^2 and gaugino masses $M_{1/2}$ at $M_{\rm Pl}$, and multinomial scalar terms proportional to terms in W_{yuk} between $M \sim M_{\rm Pl}$ and $M_{\rm GUT}$:

$$-\mathcal{L}_{soft} = M^{-1}\overline{Y}_{33}\mathbf{16}_3\mathbf{16}_3\mathbf{10}_d S_1 + \dots\ , \tag{3}$$

where now the fields stand for their scalar components. After symmetry breaking, this provides trilinear terms $\xi_{ij}L_iH_dE_j$, where

$(\xi_{33})_{\ \rm GUT} = M^{-1}\ \overline{Y}_{33}^{\rm GUT}\ \langle S_1\rangle$, *etc.*

4. NON-UNIVERSALITY AT $M_{\rm GUT}$.

There are two sources of non-universality at $M_{\rm GUT}$:

(1) There are flavor-dependent corrections δm_{16}^2 to the $\mathbf{16}_i^*\mathbf{16}_j$ scalar mass matrix. The off-diagonal elements are tied directly to the large t-quark coupling, and originate in the integration of diagrams such as Fig. 1 between $M_{\rm Pl}$ and $M_{\rm GUT}$.

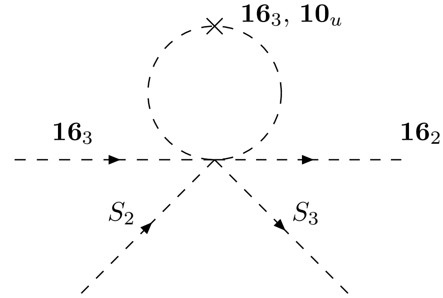

Figure 1. One loop contribution to the 3-2 entry of δm_{16}^2 .

The result of such integrations (with details given in Ref. [3]) gives a contribution to the slepton mass matrix at GUT

$$\begin{aligned} \delta m_{16}^2 &= -10\,\frac{m_{16}^2}{8\pi^2}\ \log\left(\frac{M_{\rm Pl}}{M_{\rm GUT}}\right) \\ &\cdot \begin{pmatrix} Y_{12}^2 & 0 & BC \\ 0 & Y_{12}^2 & 2BA \\ BC & 2AB & 4A^2 \end{pmatrix} \end{aligned} \tag{4}$$

Note that the third generation receives a large contribution (δm_{33}^2) from the top Yukawa A. (It turns out that $A \approx 3$.) This leads to a significant (indeed, nonperturbative) splitting of $m_{\tilde\tau}^2$ from $m_{\tilde\mu}^2 \simeq m_{\tilde e}^2$. (This possibility was noted in Ref. [6]). In the present model, flavor violations in the $\mu - e$ sector will not involve the $\tilde\tau$ or its mass. Flavor violations in $\tau - \mu$ or $\tau - e$ processes will involve loops with $\tilde\tau$, and this mass splitting will be simply parameterized when these processes are discussed.

(2) Because the SO(10) content of the Higgs structure contributing to the 2-2 matrix element of the superpotential is different from that contributing to the other entries, there will be flavor dependence induced in the $\mu - e$ sector of the effective trilinear matrix ξ_{ij} at GUT. The radiative corrections pertinent to this part of the soft-breaking are shown in Fig. 2.

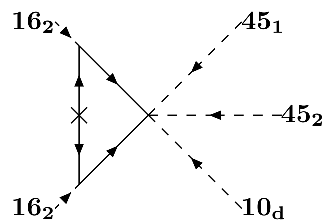

Figure 2. One-loop contribution to the effective trilinear (ξ) at GUT not in common with one-loop renormalization of effective Yukawa. Not shown are additional figures with gaugino loops for all pairs of legs.

Together with the tree contribution, they lead to the following expression for the soft trilinear matrix ξ_e at $M_{\rm GUT}$ (after SO(10) breaking):

$$\begin{aligned}(\xi^e_{ab})_G &= m_0 A_0 \begin{pmatrix} 0 & F & 0 \\ F & -3E & 0 \\ 0 & 0 & D \end{pmatrix} \\ &- 63\frac{\alpha_{\rm GUT}}{4\pi}\ M_{1/2}\ \log\left(\frac{M_{\rm Pl}}{M_{\rm GUT}}\right) \\ &\cdot \begin{pmatrix} 0 & F & 0 \\ F & (127/63)(-3E) & 0 \\ 0 & 0 & D \end{pmatrix} \end{aligned} \quad (5)$$

- We now observe that neither δm^2_{16} (Eq. (4)) nor $\delta\xi$ (second term in Eq. (5) is proportional to the Yukawa matrix, so that these will prove a source of lepton flavor violation.

5. THE INSERTIONS

On going to a lepton flavor-diagonal basis, one obtains from Eqs. (4) and (5) the slepton flavor changing insertions (ignoring $\tau - e$ mixing)

$$\begin{aligned}\Delta^{LL}_{\tau\mu} &= \Delta^{RR}_{\tau\mu} \simeq -20\ BA\ \frac{m^2_{16}}{8\pi^2}\ \log\left(\frac{M_{\rm Pl}}{M_{\rm GUT}}\right) \\ \Delta^{LR}_{\mu e} &= \Delta^{RL}_{\mu e} \simeq 64\ \frac{\alpha_{\rm GUT}}{4\pi}\sqrt{m_e m_\mu} \\ &\cdot\ M_{1/2}\ \log\left(\frac{M_{\rm Pl}}{M_{\rm GUT}}\right)\quad . \end{aligned} \quad (6)$$

As advertised, the $\mu - e$ insertion is independent of all 3rd generation physics. Moreover, it is *independent* of the MSSM μ parameter. The fact that it is non-zero and large *both* originate in the Georgi-Jarlskog constraint.[2]

6. PARAMETERS

The Feynman diagrams of Fig. (3) are then calculated, with the Δ's of Eq. (6) as insertions. The Yukawa parameters ($A - F$) at $M_{\rm GUT}$ are obtained by running to low energies and fitting to fermion masses and mixing. The rest of the parameters necessary (chargino and neutralino masses and mixing angle, and slepton masses) are obtained as functions of a two parameter space: M_G, the gaugino mass at $M_{\rm GUT}$, and $m_{\tilde\mu_L}$, the smuon (and selectron) mass at low energies. We have fixed $\tan\beta = 3,\ A_0 = 1,$ and $\mu > 0$.

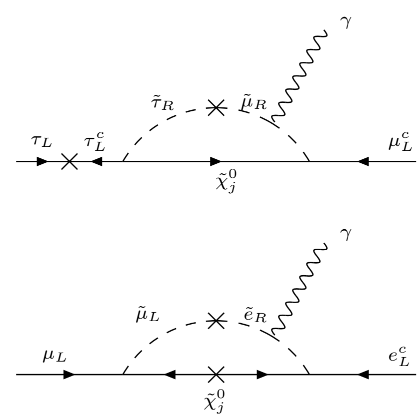

Figure 3. Top figure: one of the graphs contributing to $\tau \to \mu\gamma$. Others (with $L \leftrightarrow R$, with a $\tilde\nu_\tau$-chargino loop, and with the photon line attached to $\tilde\tau$) are not shown. Bottom figure: similarly for $\mu \to e\gamma$.

7. RESULTS

The results, displayed in Fig. (4), are insensitive to A_0 and $sgn(\mu)$, and vary only slowly with

[2] The parametric form of the $\mu \to e$ transition described in Ref. [6] is the same as in Eq. (6); however, the model used by these authors does not incorporate a realistic Yukawa texture.

$\tan\beta$ for this quantity in the range $3-10$. The quantity x referred to in the caption to Fig. (4) is a measure of the large renormalization of the $\tilde{\tau}$ mass at GUT, $m^2_{\tilde{\tau}}|_{\rm GUT} = x m^2_{16}|$.

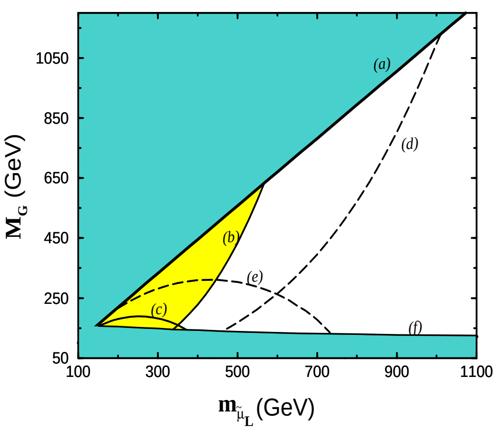

Figure 4. $(M_G, m_{\tilde{\mu}_L})$ parameter space excluded by present and (possible) future data (all curves are for $\tan\beta = 3, \mu > 0, A_0 = 1$). Area below line (f) is excluded by direct chargino search at the M_Z pole. Area above line (a) is excluded by the R-G analysis (see discussion in text). Area between lines (a) and (b) is excluded by present upper limit on $BR(\mu \to e\gamma)$. Area between line (c) $(x = 0.5)$ and axes is excluded by present upper limit on $\tau \to \mu\gamma$. Lines (d) and (e) show the range of parameters excluded if current limits were decreased by a factor of 10.

The principal results (and conclusions) which can be gleaned from Fig. (4) can be summarized as follows:

- The breaking of lepton-down quark universality at $M_{\rm GUT}$, as exemplified by the Georgi-Jarlskog texture, is most naturally implemented by imposing a Higgs structure with different representation content in the 2-2 sector.

- This can induce substantial non-universality in the $\tilde{\mu} - \tilde{e}$ sector as a result of radiative corrections between $M_{\rm Pl}$ and $M_{\rm GUT}$.

- In SO(10), this non-universality is greatly enhanced because of large group-theoretic factors. The result is manifested in a decay rate for $\mu \to e\gamma$ which impinges on present experimental limits for certain interesting regions of MSSM parameter space. If current limits were decreased by a factor of 10, a large portion of the slepton mass range would be excluded.

- Conversely, either or both of $\mu \to e\gamma$ and $\tau \to \mu\gamma$ could be observed for superpartner masses in the the few hundred GeV range at the price of this factor of 10 improvement in statistics.

- It should be stressed that physical basis underlying the results obtained here, namely a breaking of universality in the *representation content* of the 2-2 Higgs assignment, will lead to a substantial rate for $\mu \to e\gamma$ in any other SUSY-GUT model.

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